A Toy Model for Magnetic Extraction of Energy from Black Hole Accretion Disc

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Abstract

A toy model for magnetic extraction of energy from black hole (BH) accretion disk is discussed by considering the restriction of the screw instability to the magnetic field configuration. Three mechanisms of extracting energy magnetically are involved. (1) The Blandford-Znajek (BZ) process is related to the open magnetic field lines connecting the BH with the astrophysical load; (2) the magnetic coupling (MC) process is related to the closed magnetic field lines connecting the BH with its surrounding disk; and (3) a new scenario (henceforth the DL process) for extracting rotational energy from the disk is related to the open field lines connecting the disk with the astrophysical load. The expressions for the electromagnetic powers and torques are derived by using the equivalent circuits corresponding to the above energy mechanisms. It turns out that the DL power is comparable with the BZ and MC powers as the BH spin approaches unity. The radiation from a quasi-steady thin disk is discussed in detail by applying the conservation laws of mass, energy and angular momentum to the regions corresponding to the MC and DL processes. In addition, the poloidal currents and the current densities in BH magnetosphere are calculated by using the equivalent circuits.

Key words: Black holes, Physics of black holes, Infall, accretion, and accretion disks, Relativity and gravitation
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1 INTRODUCTION

As is well known, the Blandford-Znajek (BZ) mechanism has been regarded as a reasonable process for powering the radio jets in AGNs (Blandford

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& Znajek 1977; Rees 1984). Recently the BZ mechanism has been used as a central engine for powering gamma-ray bursts (GRBs), where rotating energy of a stellar black hole (BH) with magnetic field of $10^{15} gauss$ is extracted along the magnetic field lines supported by a magnetized accretion disk (Lee et al. 2000; Wang et al. 2002a). According to the BZ theory the power extracted from the BH arises from the rotation of the open magnetic field lines relative to the BH horizon, resulting in the electromotive force (EMF) in an equivalent circuit (MacDonald and Thorne 1982, hereafter MT82). The BZ power can be effectively regarded as the power of the current dissipated on the astrophysical load.

Recently much attention has been paid on the magnetic coupling (MC) process, where energy and angular momentum are transferred from a fast-rotating BH to its surrounding disk by virtue of the closed field lines connecting them (Blandford 1999; Li 2002a, hereafter L02; Wang et al. 2002b, 2003a, hereafter W02, W03a, respectively).

In W02 we worked out an equivalent circuit to calculate the BZ and MC powers. Very recently we discussed the condition for the coexistence of the BZ and MC processes (CEBZMC), and found that the state of CEBZMC always accompanies the screw instability of the magnetic field connecting a rotating BH with its surrounding disk (Wang et al. 2003b, 2004, hereafter W03b and W04, respectively). It turns out that the screw instability will occur at some place far away from the inner edge of the disk, if the BH spin and the power-law index for the variation of the magnetic field are greater than some critical values.

In this paper a new scenario for extracting rotational energy of the disk matter is proposed by considering the configuration of the magnetic field restricted by the screw instability. To facilitate description, this mechanism is referred to as the DL process, implying that energy and angular momentum are extracted magnetically from **disk** to **load**. By using another equivalent circuit we derived the expression for the DL power and torque under some assumptions on the unknown astrophysical load.

This paper is organized as follows. In §2 the restriction of the screw instability to the configuration of the magnetic field is discussed, in which the three energy mechanisms are contained. In §3 the expressions for the powers and torques of the three mechanisms are derived in the two kinds of equivalent circuits. We compare these powers with the variation of the two parameters, i.e., the BH spin and the power - law index of the magnetic field on the disk. It turns out that the DL power is generally less than the BZ and MC powers, and it is comparable with the latter two when the BH spin approaches unity. In §4 the radiation from a quasi - steady thin disk is discussed in detail by applying the conservation laws of mass, energy and angular momentum to the

regions corresponding to the MC and DL processes. In §5 the poloidal current densities flowing from the BH magnetosphere into the horizon and disk are calculated in the regions corresponding to the three energy mechanisms. Finally, in §6, we summarize our main results and argue that this model can be used to extract clean energy from a rotating BH for powering GRBs.

Throughout this paper the geometric units $G = c = 1$ are used, and the assumptions for BH accretion disk and the magnetic field are adopted as given in W03b and W04.

2 RESTRICTION OF SCREW INSTABILITY TO CONFIGURATION OF MAGNETIC FIELD

It is well known that the magnetic field configurations with both poloidal and toroidal components can be screw instable (Kadomtsev 1966; Bateman 1978). According to the Kruskal-Shafranov criterion (Kadomtsev 1966), the screw instability will occur, if the toroidal magnetic field becomes so strong that the magnetic field line turns around itself about once. Recently some authors discussed the screw instability with different conditions in the BH magnetosphere (Gruzinov 1999; Li 2000a; Tomimatsu et el. 2001). In W04 we discussed the screw instability of the magnetic field in the MC process, and argued that the instability will occur if the following criterion is satisfied,

$$(2\pi\varpi_{_D}/L)\, B_D^p \Big/ B_D^T < 1, \qquad (1)$$

where L is the poloidal length of the closed field line connecting the BH with the disk, B_D^p and B_D^T are the poloidal and toroidal components of the magnetic field on the disk, respectively, and $\varpi_{_D}$ is the cylindrical radius on the disk and reads

$$\varpi_{_D} = \xi M \chi_{ms}^2 \sqrt{1 + a_*^2 \xi^{-2} \chi_{ms}^{-4} + 2a_*^2 \xi^{-3} \chi_{ms}^{-6}}. \qquad (2)$$

In equation (2) M and a_* are the BH mass and spin, respectively. The parameter $\xi \equiv r/r_{ms}$ is the radial coordinate on the disk, which is defined in terms of the radius of the marginally stable orbit, and r_{ms} is related to M and a_* by the following relation (Novikov & Thorne 1973),

$$\left.\begin{array}{l}
r_{ms} \equiv M\chi_{ms}^2, \\
\chi_{ms} = \left\{3 + A_2 \pm \left[(3 - A_1)(3 + A_1 + 2A_2)\right]^{1/2}\right\}^{1/2}, \\
A_1 = 1 + (1 - a_*^2)^{1/3}\left[(1 + a_*)^{1/3} + (1 - a_*)^{1/3}\right], \\
A_2 = (3a_*^2 + A_1^2)^{1/2}
\end{array}\right\} \qquad (3)$$

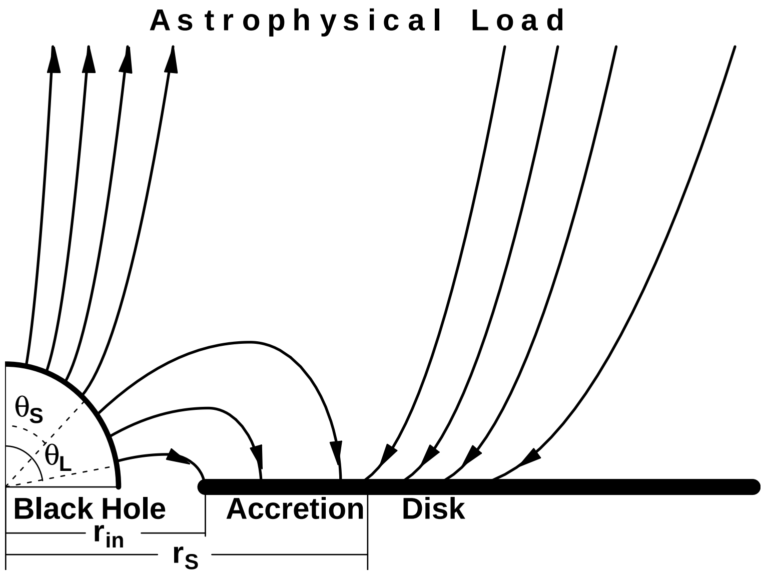


Fig. 1. Configuration of the magnetic field restricted by the screw instability

It is found, from the criterion (1), that the screw instability will occur, provided that the BH spin a_* and the power-law index n are great enough. For the given values of a_* and n we can determine the disk region for the screw instability by using the criterion (1) as follows,

$$\xi_S < \xi < \infty, \tag{4}$$

where $\xi_S \equiv r_S/r_{ms}$ is the minimum radial coordinate for the screw instability. We think that the screw instability prevents the closed field lines to reach the region indicated by inequality (4). It seems reasonable that the configuration of the magnetic field in this region might consist of the open field lines connecting the disk with the astrophysical load, and this region is referred to as the DL region corresponding to the DL process. Considering the restriction of the screw instability, we modify the configuration of the magnetic field as shown in Figure 1, and the three mechanisms of extracting energy magnetically from the BH accretion disk are included: (1) the BZ process involved the open field lines connecting the BH with the astrophysical load, (2) the MC process involved the closed field lines connecting the BH with the surrounding disk, and (3) the DL process involved the open field lines connecting the disk with the astrophysical load.

Now we give a brief description for the configuration in Figure 1. In W03b we discussed the angular boundary between the open and closed field lines on the BH horizon. It is shown that the boundary angle θ_M exists in CEBZMC, if the parameters a_* and n are great enough. In W04 we argued that the state of CEBZMC always accompanies the screw instability, and the minimum radial coordinate $\xi_S \equiv r_S/r_{ms}$ for the screw instability can be determined by the

criterion (1). Considering the restriction of the screw instability to the closed field lines and the action of the magnetic pressure on the horizon, we think that the boundary angle will be extended from θ_M to θ_S. The angles θ_M and θ_S are related respectively to infinitive and the minimum radial coordinate ξ_S by the mapping relation (W04):

$$\cos\theta - \cos\theta_L = \int_1^{\xi} \mathrm{G}\left(a_*; \xi, n\right) d\xi, \tag{5}$$

where

$$\mathrm{G}\left(a_*; \xi, n\right) = \frac{\xi^{1-n}\chi_{ms}^2\sqrt{1 + a_*^2\chi_{ms}^{-4}\xi^{-2} + 2a_*^2\chi_{ms}^{-6}\xi^{-3}}}{2\sqrt{\left(1 + a_*^2\chi_{ms}^{-4} + 2a_*^2\chi_{ms}^{-6}\right)\left(1 - 2\chi_{ms}^{-2}\xi^{-1} + a_*^2\chi_{ms}^{-4}\xi^{-2}\right)}}. \tag{6}$$

Therefore the angular region of the open field lines for the BZ process is given by

$$0 < \theta < \theta_S, \tag{7}$$

and the angular region of the closed field lines for the MC process is given by

$$\theta_S < \theta < \theta_L, \tag{8}$$

where θ_L is the lower boundary angle of the closed field lines. Throughout this paper $\theta_L = 0.45\pi$ is assumed in calculations. Accordingly the value range of the radial coordinate for the MC process is given by

$$1 < \xi < \xi_S, \tag{9}$$

where $\xi = 1$ and ξ_S correspond to θ_L and θ_S by the mapping relation (5), respectively.

Following Blandford (1976), we assume that the poloidal magnetic field varies with the parameter ξ on the disk as a power law (W03a, W03b),

$$B_D^p = B_H^p\left[r_H/\varpi_D\left(r_{ms}\right)\right]\xi^{-n}, \quad 1 < \xi < \xi_S, \tag{10}$$

where B_H^p is the poloidal component of the magnetic field on the horizon, and n is the power-law index of B_D^p varying with ξ. The quantities r_H and $\varpi_D\left(r_{ms}\right)$ are the radius of the horizon and that of the inner edge of the disk, respectively.

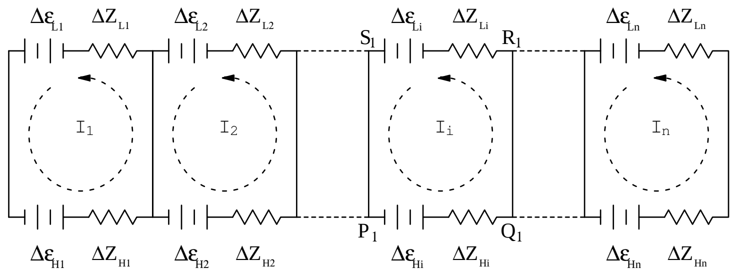


Fig. 2. Equivalent circuit for the BZ and MC processes (circuit I)

According to the above discussion the value range of the radial coordinate for the DL process is given by equation (4), and the corresponding poloidal magnetic field varies with the parameter ξ as follows,

$$B_D^p = B_H^p \left(r_{_H} / r_{_S} \right) \left(\xi / \xi_S \right)^{-n}, \quad \xi_S < \xi < \infty. \tag{11}$$

Equation (11) is similar to equation (10), where $\varpi_{_D}(r_{ms})$ and $\xi_{in} = 1$ are replaced by $r_{_S}$ and ξ_S, respectively.

3 POWER OF MAGNETIC EXTRACTION FROM BH ACCRETION DISK

It is tempting for us to discuss the power extracted from BH accretion disk based on the above magnetic field configuration. In W02 we derived the expressions for the BZ and MC powers by using an equivalent circuit as shown in Figure 2 (henceforth circuit I), where segments P_1S_1 and Q_1R_1 represent the two adjacent magnetic surfaces, and segments P_1Q_1 and R_1S_1 represent the BH horizon and the loads (either the astrophysical load or the disk load) sandwiched by the two magnetic surfaces, respectively.

By using circuit I we derive the expressions for the BZ and MC powers and those for the BZ and MC torques corresponding to the modified configuration of the magnetic field as follows,

$$P_{BZ}/P_0 = 2a_*^2 \int_0^{\theta} \frac{k(1-k)\sin^3\theta d\theta}{2-(1-q)\sin^2\theta}, \quad 0 < \theta < \theta_S \tag{12}$$

$$T_{BZ}/T_0 = 4a_*(1+q) \int_0^{\theta_S} \frac{(1-k)\sin^3\theta d\theta}{2-(1-q)\sin^2\theta}, \quad 0 < \theta < \theta_S \tag{13}$$

$$P_{MC}/P_0 = 2a_*^2 \int\limits_{\theta_S}^{\theta} \frac{\beta\left(1-\beta\right)\sin^3\theta d\theta}{2-\left(1-q\right)\sin^2\theta}, \quad \theta_S < \theta < \theta_L \tag{14}$$

$$T_{MC}/T_0 = 4a_*\left(1+q\right) \int\limits_{\theta_S}^{\theta} \frac{\left(1-\beta\right)\sin^3\theta d\theta}{2-\left(1-q\right)\sin^2\theta}, \quad \theta_S < \theta < \theta_L \tag{15}$$

where ΔP_{BZ}, ΔT_{BZ}, ΔP_{MC} and ΔT_{MC} are related by

$$\Delta P_{BZ} = \Omega_F \Delta T_{BZ}, \quad \Delta P_{MC} = \Omega_D \Delta T_{MC}. \tag{16}$$

In equations (12)—(15) the parameters k and β are respectively the ratios of the angular velocities of the open and closed field lines to the angular velocity of the horizon and read

$$k \equiv \Omega_F/\Omega_H, \quad \beta \equiv \Omega_D/\Omega_H = \frac{2\left(1+q\right)}{a_*}\left[\left(\sqrt{\xi}\chi_{ms}\right)^3 + a_*\right]^{-1}. \tag{17}$$

where Ω_F is the angular velocity of the magnetic field line, Ω_H and Ω_D are respectively the angular velocities of the horizon and the disk and read

$$\Omega_H = \frac{a_*}{2r_H}, \quad \Omega_D = \frac{1}{M(\xi^{3/2}\chi_{ms}^3 + a_*)}. \tag{18}$$

Usually $k = 0.5$ is taken for the optimal BZ power (MT82), while β depends on the BH spin and the place where the field line penetrates on the disk. In equations (12)—(15) we use the parameter $q \equiv \sqrt{1-a_*^2}$, and the parameters P_0 and T_0 are defined as

$$\begin{cases} P_0 \equiv \left(B_H^p\right)^2 M^2 \approx B_4^2 \left(M/M_\odot\right)^2 \times 6.59 \times 10^{28} erg \cdot s^{-1}, \\ T_0 \equiv \left(B_H^p\right)^2 M^3 \approx B_4^2 \left(M/M_\odot\right)^3 \times 3.26 \times 10^{23} g \cdot cm^2 \cdot s^{-2}, \end{cases} \tag{19}$$

where B_4 is the strength of the poloidal magnetic field on the horizon in units of 10^4 *gauss*.

Considering that the poloidal magnetic field will exert a braking torque on the current in the DL region, we can calculate the DL power and torque by using the equivalent circuit as shown in Figure 3. Henceforth this equivalent circuit is referred to as circuit II, where segments P_2S_2 and Q_2R_2 represent the two adjacent magnetic surfaces consisting of the open field lines in the DL region, and segments P_2Q_2 and R_2S_2 represent the disk surface in the

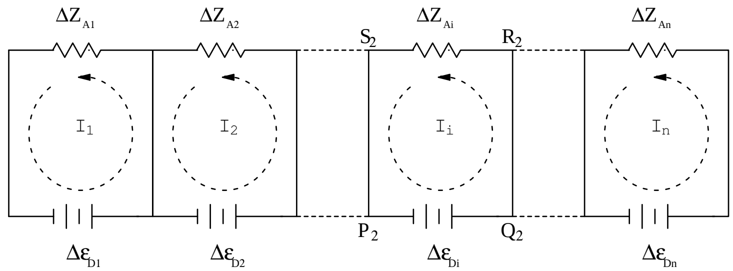


Fig. 3. Equivalent circuit for the DL process (circuit II)

DL region and the load sandwiched by the two adjacent magnetic surfaces, respectively. The quantities ΔZ_A and $\Delta\varepsilon_D$ (the subscript "i" is omitted) are the resistance of the load and EMF due to the rotation of the disk, respectively. The disk load is neglected in calculations by considering the perfect conductivity of plasma.

The following equations are used to derive the DL power in circuit II, which are similar to those given in deriving the BZ power and MC power in W02.

$$\Delta P_{DL} = I_{DL}^2 \Delta Z_A, \quad I_{DL} = \Delta\varepsilon_D / \Delta Z_A, \quad \Delta\varepsilon_D = -\left(\Delta\Psi_D / 2\pi\right)\Omega_D, \tag{20}$$

where I_{DL} is the current in each loop of circuit II. The minus sign in $\Delta\varepsilon_D$ arises from the direction of the magnetic flux between the two adjacent magnetic surfaces, which is given by

$$\Delta\Psi_D = B_D^P 2\pi \left(\varpi\rho \big/ \sqrt{\Delta}\right)_{\theta=\pi/2} dr. \tag{21}$$

The concerned Kerr metric coefficients are given by (Thorne, Price & MacDonald 1986)

$$\begin{cases} \varpi = (\Sigma/\rho)\sin\theta, \\ \Sigma^2 \equiv \left(r^2 + a^2\right)^2 - a^2\Delta\sin^2\theta, \\ \rho^2 \equiv r^2 + a^2\cos^2\theta, \\ \Delta \equiv r^2 + a^2 - 2Mr. \end{cases} \tag{22}$$

Since the astrophysical load remains unknown, we give some simplified assumptions as follows.

(1) The load is axisymmetric, being located evenly in a plane $\boldsymbol{P}$ with some height above the disk.

(2) Each open field line intersects with the disk and the plane $\boldsymbol{P}$ at the cylindrical radii r and r', respectively. The both radii are related by

$$r' = \lambda r, \tag{23}$$

where λ is assumed to be a constant.

(3) The surface resistivity $\sigma_{_L}$ of the unknown load is assumed to obey the following relation,

$$\sigma_{_L} = \alpha_{_Z} \sigma_{_H} = 4\pi \alpha_{_Z}, \tag{24}$$

where $\alpha_{_Z}$ is a parameter, and $\sigma_{_H} = 4\pi = 377\ ohm$ is the surface resistivity of the BH horizon. Throughout this paper $\alpha_{_Z} = 1$ is assumed in calculations. The load resistance ΔZ_A between the two adjacent magnetic surfaces can be written as

$$\Delta Z_A = \sigma_{_L} \frac{dr'}{2\pi r'} = 2\alpha_{_Z} \frac{dr}{r}, \tag{25}$$

where equations (23) and (24) are used in the last step. Incorporating equation (11) with equations (20)—(25), we have

$$\Delta P_{DL}/P_0 = f\left(a_*, \xi, n\right) d\xi, \tag{26}$$

where the function $f\left(a_*, \xi, n\right)$ is expressed by

$$f(a_*, \xi, n) = \frac{\left(1+q\right)^2 \left(1 + a_*^2 \chi_{ms}^{-4} \xi^{-2} + 2a_*^2 \chi_{ms}^{-6} \xi^{-3}\right) \chi_{ms}^4 \xi_S \left(\xi/\xi_S\right)^{-2n+3}}{2\left(\xi^{3/2} \chi_{ms}^3 + a_*\right)^2 \left(1 - 2\chi_{ms}^{-2} \xi^{-1} + a_*^2 \chi_{ms}^{-4} \xi^{-2}\right)}. \tag{27}$$

Similarly, the DL power is related to the DL torque by

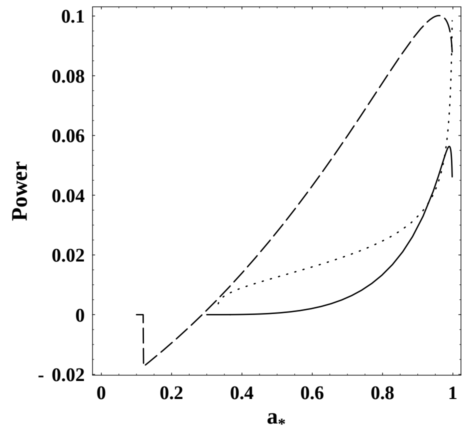
$$\Delta P_{DL} = \Omega_D \Delta T_{DL}, \tag{28}$$

and we have

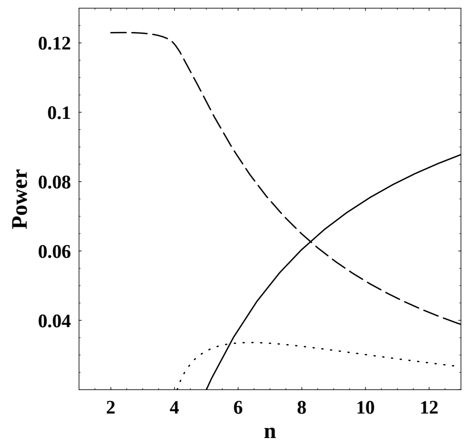
$$\Delta T_{DL}/T_0 = g\left(a_*, \xi, n\right) d\xi, \tag{29}$$

where the function $g\left(a_*, \xi, n\right)$ is expressed by

$$g(a_*, \xi, n) = \frac{\left(1+q\right)^2 \left(1 + a_*^2 \chi_{ms}^{-4} \xi^{-2} + 2a_*^2 \chi_{ms}^{-6} \xi^{-3}\right) \chi_{ms}^4 \xi_S \left(\xi/\xi_S\right)^{-2n+3}}{2\left(\xi^{3/2} \chi_{ms}^3 + a_*\right) \left(1 - 2\chi_{ms}^{-2} \xi^{-1} + a_*^2 \chi_{ms}^{-4} \xi^{-2}\right)}. \tag{30}$$



(a)



(b)

Fig. 4. The BZ power (solid line), the MC power (dashed line) and the DL power (dotted line) (a) versus the BH spin a_* with $n = 5.5$, and (b) versus the power-law index n with $a_* = 0.9$.

Integrating equations (26) and (29) over the DL region, we have the DL power and torque expressed by

$$P_{DL}(a_*, \xi, n)/P_0 = \int_{\xi_S}^{\xi} f(a_*, \xi', n) d\xi', \tag{31}$$

$$T_{DL}\left(a_*, \xi, n\right)/T_0 = \int_{\xi_S}^{\xi} g\left(a_*, \xi', n\right) d\xi'. \tag{32}$$

For the given values of a_* and n the strength of P_{BZ}/P_0, P_{MC}/P_0 and P_{DL}/P_0 are compared by using equations (12), (14) and (31) as shown in Figures 4.

From Figure 4 we obtain the following results,

(1) For the increasing a_* with the given n, both the BZ and the MC powers vary non-monotonically, attaining their maxima as the BH spin approaches unity, while the DL power increases monotonically.

(2) For the increasing n with the given a_*, the BZ power increases monotonically, the MC power decreases monotonically, while the DL power varies non-monotonically, attaining its maximum value $P_{DL} \approx 0.034$ with $n = 6.39$.

(3) The MC power is generally greater than the BZ power, if the power-law index n is not very big. The DL power is generally less than the BZ and MC powers, and it is comparable with the latter two powers as the BH spin approaches unity.

4 RADIATION FROM A MAGNETIZED ACCRETION DISK

4.1 Radiation from the MC region

Radiation from a relativistic steady thin disk around a Kerr BH has been discussed based on the three conservation laws of mass, energy and angular momentum with the "no-torque boundary condition" by some authors (Novikov & Thorne 1973, Page & Thorne 1974). By combining the MC effects with the above conservation laws and the "no-torque boundary condition", the following equation of radiation from a relativistic quasi-steady thin disk was derived in L02,

$$F_{MC}^{total} = F_{DA}^{I} + F_{MC}, \tag{33}$$

where F_{DA}^{I} and F_{MC} are the radiation fluxes due to disk accretion and the MC effects, respectively, and they are expressed by

$$F_{DA}^{I} = \frac{\dot{M}_D}{4\pi r} \frac{(-\partial\Omega_D/\partial r)}{(E^+ - \Omega_D L^+)^2} \int_{r_{ms}}^{r} \left(E^+ - \Omega_D L^+\right) \left(\partial L^+ / \partial r\right) dr, \tag{34}$$

$$F_{MC} = \frac{(-\partial\Omega_D/\partial r)}{r\,(E^+ - \Omega_D L^+)^2} \int_{r_{ms}}^{r} \left(E^+ - \Omega_D L^+\right) r H_{MC} dr. \tag{35}$$

In equation (35) H_{MC} is the flux of angular momentum transferred between the BH and the disc by the magnetic field, and E^+ and L^+ are the specific

energy and angular momentum of a particle in the disc, respectively, and they read (Novikov & Thorne 1973)

$$E^{+} = \left(1 - 2\chi^{-2} + a_* \chi^{-3}\right) \Big/ \left(1 - 3\chi^{-2} + 2a_* \chi^{-3}\right)^{1/2}, \tag{36}$$

$$L^{+} = M\chi\left(1 - 2a_* \chi^{-3} + a_*^2 \chi^{-4}\right) \Big/ \left(1 - 3\chi^{-2} + 2a_* \chi^{-3}\right)^{1/2}. \tag{37}$$

From equations (36) and (37) we have $E^{+} = E_{ms}$ and $L^{+} = L_{ms}$ for $\xi = 1$ with $\chi = \chi_{ms}$. The expression for H_{MC} can be worked out by using the following relation,

$$\partial T_{MC}/\partial r = \frac{\left(\partial T_{MC}/\partial\theta\right)\left(d\theta/d\xi\right)}{\xi M \chi_{ms}^2} = 4\pi r H_{MC}, \tag{38}$$

where $\partial T_{MC}/\partial\theta$ and $d\theta/d\xi$ can be calculated using equations (15) and (5). In W03a we expressed H_{MC} as follows,

$$H_{MC}/H_0 = A\left(a_*, \xi\right)\xi^{-n}, \qquad 1 < \xi < \xi_S, \tag{39}$$

where

$$H_0 = \left(B_H^p\right)^2 M = 1.48 \times 10^{13} \times B_4^2 \left(M/M_\odot\right)\ g \cdot s^{-2}, \tag{40}$$

$$\begin{cases} A\left(a_*, \xi\right) = \frac{a_*(1-\beta)(1+q)}{2\pi\chi_{ms}^2\left[2\csc^2\theta - (1-q)\right]} F_H\left(a_*, \xi\right), \\ \\ F_H\left(a_*, \xi\right) = \frac{\sqrt{1 + a_*^2\chi_{ms}^{-4}\xi^{-2} + 2a_*^2\chi_{ms}^{-6}\xi^{-3}}}{\sqrt{\left(1 + a_*^2\chi_{ms}^{-4} + 2a_*^2\chi_{ms}^{-6}\right)\left(1 - 2\chi_{ms}^{-2}\xi^{-1} + a_*^2\chi_{ms}^{-4}\xi^{-2}\right)}}. \end{cases} \tag{41}$$

Since the magnetic field on the horizon is brought and held by its surrounding magnetized disk, there must exist some relations between the magnetic field and the accretion rate. As a matter of fact these relations might be rather complicated, and would be very different in different situations. One of them is given by considering the balance between the pressure of the magnetic field on the horizon and the ram pressure of the innermost parts of an accretion flow (Moderski, Sikora & Lasota 1997), i.e.,

$$\left(B_H^p\right)^2 \Big/ (8\pi) = P_{ram} \sim \rho c^2 \sim \dot{M}_D \Big/ \left(4\pi r_H^2\right), \tag{42}$$

From equation (42) we define F_0 as

$$F_0 \equiv \left(B_H^p\right)^2 = \frac{2\dot{M}_D}{M^2\left(1+q\right)^2}. \tag{43}$$

By using equations (34) and (35) we have the radiation fluxes, $F_{DA}^{I}\Big/F_0$ and F_{MC}/F_0, versus the radial parameter ξ for the given values of the power-law index and the BH spin as shown in Figure 5.

Expecting Figure 5 we find that the peak of F_{MC}/F_0 is not only closer to the inner edge of the disk, but also is greater than that of $F_{DA}^{I}\Big/F_0$. Furthermore, F_{MC}/F_0 varies more steeply with ξ than $F_{DA}^{I}\Big/F_0$ does. Thus the radiation flux due to the MC mechanism can result in a very steep emissivity in the inner region of the disk, which is consistent with the recent *XMM-Newton* observation of the nearby bright Seyfert 1 galaxy MCG-6-30-15 (Wilms et al. 2001; Li 2002b; W03a).

4.2 Radiation from the DL region

We can show that the radiation flux from the DL region also consists of two terms:

$$F_{DL}^{Total} = F_{DA}^{II} + F_{DL}, \tag{44}$$

where F_{DL} is the electromagnetic flux in the DL process, and F_{DA}^{II} is the radiation flux due to disk accretion in the DL region. The flux F_{DL} can be worked out by

$$F_{DL} = \frac{\Delta P_{DL}}{2\pi r \Delta r}. \tag{45}$$

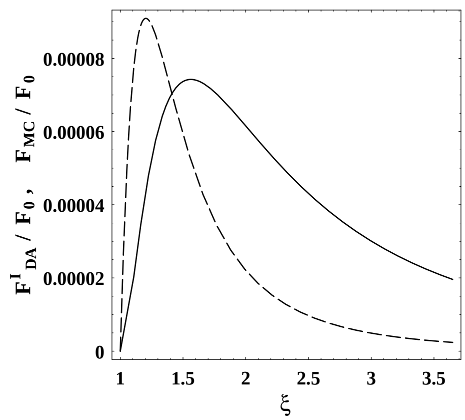
Substituting equation (26) into equation (45), we have

$$F_{DL}/F_0 = \frac{f\left(a_*, \xi, n\right)}{2\pi\xi\chi_{ms}^4} \tag{46}$$

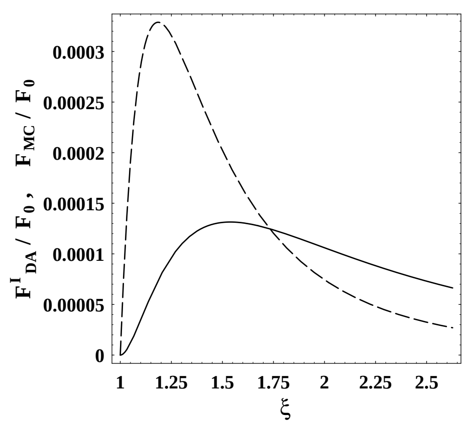
The difference between the radiation from the DL region and that from the MC region lies in two aspects.

(1) The outgoing flux F_{MC} can be obtained after resolving the equations of the conservation laws with P_{MC} and T_{MC} incoming the disk, while the outgoing flux F_{DL} is given before resolving the equations from the conservation laws;

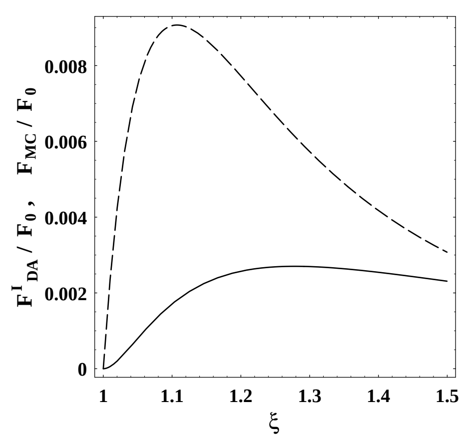
(2) "No-torque boundary condition" can be used at r_{ms} for the solution in the MC region, while this boundary condition is not valid at r_S for the solution in the DL region.



(a)



(b)



(c)

Fig. 5. The radiation flux from the MC region, F^I_{DA}/F_0 (solid line) and F_{MC}/F_0 (dashed line) versus ξ for $1 < \xi < \xi_S$ with $n = 5.5$ and different values of the BH spin: (a) $a_* = 0.5$, (b) $a_* = 0.7$ and (c) $a_* = 0.998$.

Thus we find the flux F_{DA}^{II} by applying the conservation laws of energy and angular momentum to the DL region, i.e.,

$$\frac{\partial}{\partial r}\left(\dot{M}_D E^+ - 2\pi r W_\varphi^r \Omega_D\right) = 4\pi r\left(F_{DA}^{II}E^+ + H_{DL}\Omega_D\right), \tag{47}$$

$$\frac{\partial}{\partial r}\left(\dot{M}_D L^+ - 2\pi r W_\varphi^r\right) = 4\pi r\left(F_{DA}^{II}L^+ + H_{DL}\right), \tag{48}$$

where accretion rate $\dot{M}_D$ is regarded as a constant for a quasi-steady disk, and W_φ^r is the internal viscous torque per unit circumference. Incorporating equations (47) and (48), we have

$$W_\varphi^r = \frac{2\left(E^+ - \Omega_D L^+\right)}{\left(-d\Omega_D/dr\right)}F_{DA}^{II}. \tag{49}$$

The quantity H_{DL} is the flux of angular momentum, which is related to the electromagnetic torque by

$$\partial T_{DL}/\partial r = \left(\partial T_{DL}/\partial\xi\right)\left(\partial\xi/\partial r\right) = 4\pi r H_{DL}. \tag{50}$$

Substituting equation (32) into equation (50), we have

$$H_{DL}/H_0 = g(a_*,\xi,n)\Big/\left(4\pi\xi\chi_{ms}^4\right). \tag{51}$$

Substituting equation (49) into equation (48) and integrating equation (48) over the DL region, we have the expression for F_{DA}^{II} as follows,

$$F_{DA}^{II} = F_A + F_B + F_C, \tag{52}$$

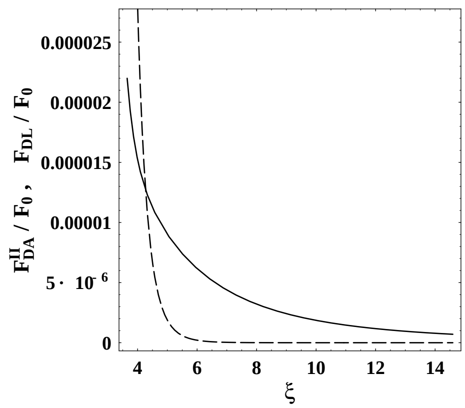
where

$$F_A = \frac{\dot{M}_D}{4\pi r}\frac{\left(-\partial\Omega_D/\partial r\right)}{\left(E^+ - \Omega_D L^+\right)^2}\int_{r_S}^{r}\left(E^+ - \Omega_D L^+\right)\left(\partial L^+\big/\partial r\right)dr, \tag{53}$$

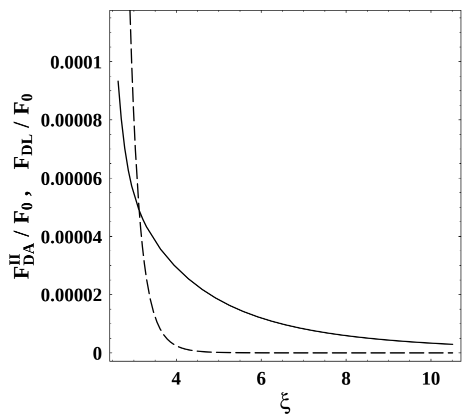
$$F_B = -\frac{\left(-\partial\Omega_D/\partial r\right)}{r\left(E^+ - \Omega_D L^+\right)^2}\int_{r_S}^{r}\left(E^+ - \Omega_D L^+\right)rH_{DL}dr, \tag{54}$$

$$F_C = \frac{-\partial\Omega_D/\partial r}{r(E^+ - \Omega_D L^+)^2}\left[\frac{(E^+ - \Omega_D L^+)^2 r F_{MC}^{Total}}{-\partial\Omega_D/\partial r}\right]_{r=r_S^-} \tag{55}$$

Expecting equations (52)—(55), we find the following characteristics of F_{DA}^{II}:



(a)



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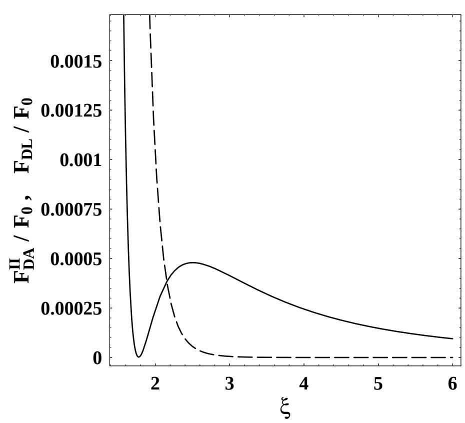


(c)

Fig. 6. The radiation flux from the DL region, F_{DA}^{II}/F_0 (solid line) and F_{DL}/F_0 (dashed line) versus ξ for $\xi_S < \xi < 4\xi_S$ with $n = 5.5$ and different values of the BH spin: (a) $a_* = 0.5$, (b) $a_* = 0.7$ and (c) $a_* = 0.998$.

(1)The flux F_A has the same form as F_{DA}^{I} except the integral region;

(2)The flux F_B has a similar form to F_{MC} except the minus sign in equation (54), which implies the contribution of the DL torque on F_{DA}^{II} is negative;

(3)The flux F_C represents the effect of the boundary condition at $r = r_S^-$, where "no-torque boundary condition" is not valid.

By using equations (52) and (46) we have the radiation fluxes, F_{DA}^{II}/F_0 and F_{DL}/F_0, versus the radial parameter ξ for the given values of the power-law index and the BH spin as shown in Figures 6.

Inspecting Figure 6, we have the following results.

(1) The radiation flux F_{DL}/F_0 decreases monotonously and very steeply with the increasing ξ. This result arises from two aspects, i.e., both the angular velocity decreases and the area of the ring from r to $r + dr$ increases with the increasing ξ.

(2) The radiation flux $F_{DA}^{II}\Big/F_0$ generally decreases monotonously with the increasing ξ, while it varies non-monotonously with ξ as the BH spin approaches unity as shown in Figure 6c. This result arises from the conservation laws of energy and angular momentum and the behavior of F_{DL}/F_0 near the boundary at r_S.

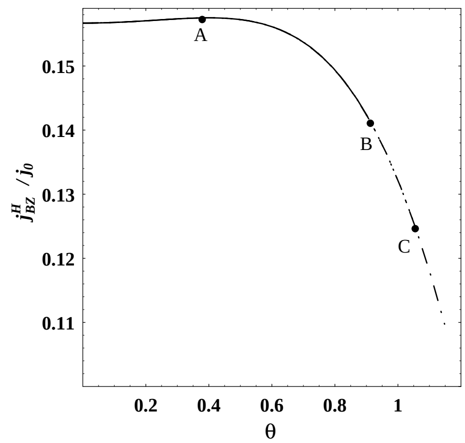
5 CURRENT DENSITIES FLOWING FROM MAGNETOSPHERE INTO HORIZON AND DISK

In W03b we calculated the current density flowing from the BH magnetosphere into the MC region on the horizon in the state of CEBZMC by using the circuit I. In this paper the three energy mechanisms of the magnetic extraction are described by using the circuits I and II, based on which the current densities flowing from the BH magnetosphere into the horizon and the disk can be calculated and compared. As argued in W03b the poloidal current flowing in each loop of circuit I can be written as

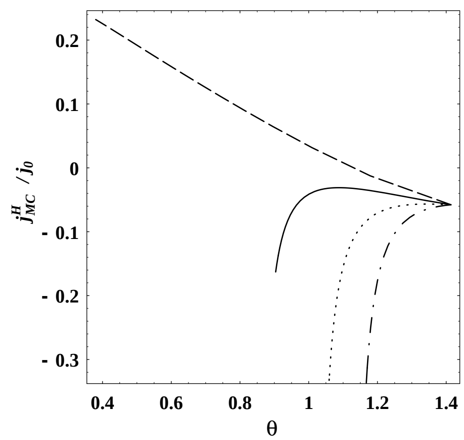
$$I_{BZ}^{H}(a_*, \theta) = I_0 \frac{a_*(1-k)}{2\csc^2\theta - (1-q)}, \quad 0 < \theta < \theta_S, \tag{56}$$

$$I_{MC}^{H}(a_*, \theta, n) = I_0 \frac{a_*(1-\beta)}{2\csc^2\theta - (1-q)}, \quad \theta_S < \theta < \theta_L, \tag{57}$$

where $I_0 = B_H^p M \approx 1.48 \times 10^{10} B_4 (M/M_\odot)\, A$. The currents I_{BZ}^{H} and I_{MC}^{H} are respectively the poloidal currents flowing on the BZ and MC regions of the



(a)



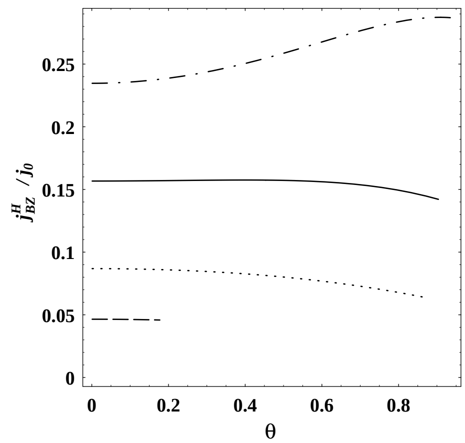
(b)

Fig. 7. Current densities versus θ with $a_* = 0.9$, $\theta_L = 0.45\pi$, (a) j^H_{BZ}/j_0 for $0 < \theta < \theta_S$, where A, B, C are the end points of the curves corresponding to n=4, 5.5, 7, respectively. (b) j^H_{MC}/j_0 for $\theta_S < \theta < \theta_L$ for $n = 4$, 5.5, 7 and 9 in dashed, solid, dotted and dot-dashed lines, respectively.

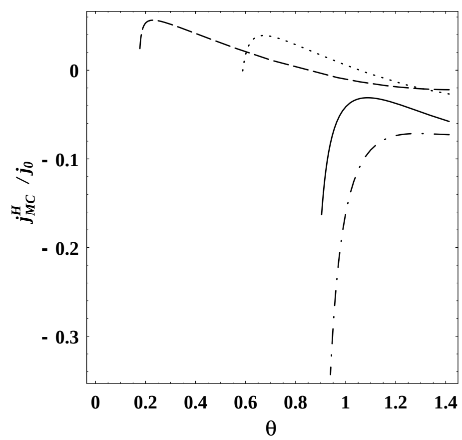
horizon, which depend on the angular coordinate θ for the given values of a_* and n. Inspecting equations (56) and (57), we find that I^H_{BZ} are generally not equal to I^H_{MC} for $\beta \neq k$. The conservation of the current at θ_S is guaranteed by the current flowing between the magnetosphere and the horizon. The continuity of the current at the boundary between the BZ and MC regions of the horizon is discussed in W03b.

Based on the conservation of current, the current densities flowing from the BH magnetosphere into the above regions of the horizon are expressed by

$$\begin{aligned} j^H_{BZ}(a_*,\theta) &= \frac{1}{2\pi(\varpi\rho)_{r=r_H}}\frac{dI^H_{BZ}}{d\theta} \\ &= j_0\frac{4(1-k)M\Omega_H\cos\theta}{\left[2-\sin^2\theta(1-q)\right]^2}, \qquad 0<\theta<\theta_S, \end{aligned} \tag{58}$$



(a)



(b)

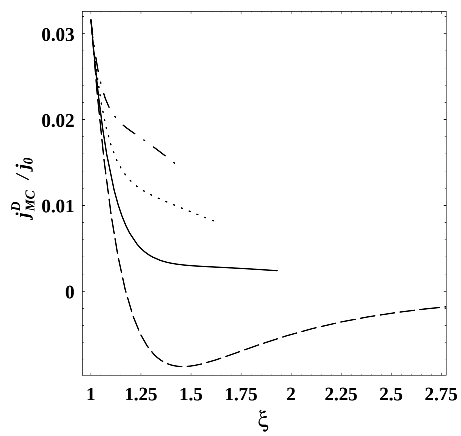
Fig. 8. Current densities versus θ with $n = 5.5$, $\theta_L = 0.45\pi$, (a) j^H_{BZ}/j_0, (b) j^H_{MC}/j_0 for a_* =0.36 , 0.62, 0.9 and 0.998 in dashed, dotted, solid, and dot-dashed lines, respectively.

$$\begin{aligned} j^H_{MC}\left(a_*, \theta, n\right) &= \tfrac{1}{2\pi(\varpi\rho)_{r=r_H}} \tfrac{dI^H_{MC}}{d\theta} \\ &= j_0 \tfrac{M\Omega_H}{2-\sin^2\theta(1-q)} \left[\tfrac{4(1-\beta)\cos\theta}{2-\sin^2\theta(1-q)} - \sin\theta\tfrac{d\beta}{d\theta}\right], \qquad \theta_S < \theta < \theta_L, \end{aligned} \tag{59}$$

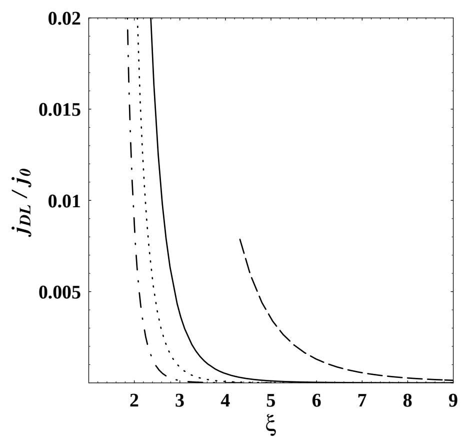
where $j_0 \equiv B^p_H/(2\pi M) = 0.108 \times B_4\left(M/M_\odot\right)^{-1} A\cdot cm^{-2}$. For the given value of a_* and n, both j^H_{BZ} and j^H_{MC} vary with angle θ in the range $0 < \theta < \theta_S$ and $\theta_S < \theta < \theta_L$, respectively.

According to circuit I the poloidal current flowing on the MC region of the disk is equal to that flowing on the MC region of the horizon, i.e.,

$$I^D_{MC}\left(a_*, \xi, n\right) = I^H_{MC} = I_0 \frac{a_*\left(1-\beta\right)}{2\csc^2\theta - (1-q)}, \quad 1 < \xi < \xi_S. \tag{60}$$



(a)



(b)

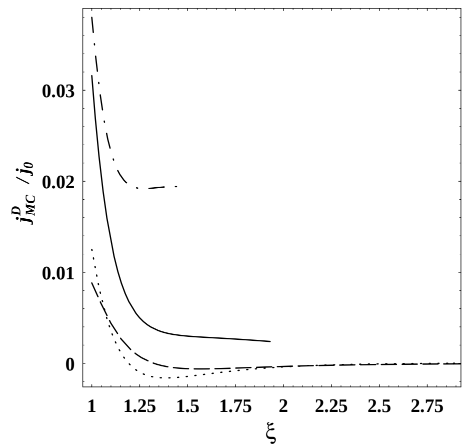
Fig. 9. Current densities versus ξ with $a_* = 0.9$, $\theta_L = 0.45\pi$, (a) j^D_{MC}/j_0, (b) j_{DL}/j_0 for n =4, 5.5, 7 and 9 in dashed, solid, dotted and dot-dashed lines, respectively.

Therefore the current density flowing from the BH magnetosphere into the disk is

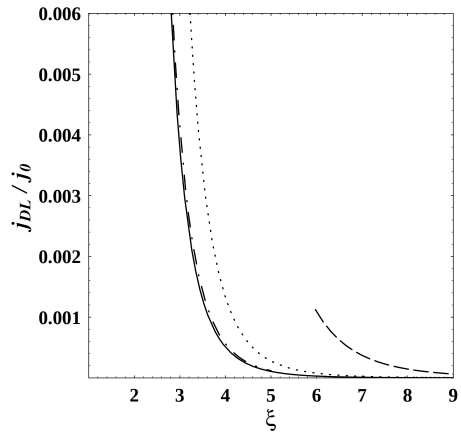
$$j^D_{MC}\left(a_*,\xi,n\right) = \frac{1}{2\pi\left(\varpi\rho\big/\sqrt{\Delta}\right)_{\theta=\pi/2}}\frac{dI^D_{MC}}{dr}, \quad 1<\xi<\xi_S. \tag{61}$$

Substituting equation (60) into equation (61), we have the current density flowing from the magnetosphere into the MC region of the disk as follows,

$$j^D_{MC}\left(a_*,\xi,n\right) = \frac{j_0 a_* C(a_*,\xi)}{2\csc^2\theta-(1-q)}\left[\frac{4(1-\beta)\csc^2\theta\cot\theta}{2\csc^2\theta-(1-q)}\frac{d\theta}{d\xi}-\frac{d\beta}{d\xi}\right],\ 1<\xi<\xi_S, \tag{62}$$



(a)



(b)

Fig. 10. Current densities versus ξ with $n = 5.5$, $\theta_L = 0.45\pi$, (a) j^D_{MC}/j_0, (b) j_{DL}/j_0 for $a_* = 0.36$, 0.62, 0.9 and 0.998 in dashed, dotted, solid and dot-dashed lines, respectively.

where

$$C\left(a_*,\xi\right) \equiv \frac{\sqrt{1+\xi^{-2}\chi_{ms}^{-4}a_*^2 - 2\xi^{-1}\chi_{ms}^{-2}}}{\xi\chi_{ms}^4\sqrt{1+\xi^{-2}\chi_{ms}^{-4}a_*^2+2\xi^{-3}\chi_{ms}^{-6}a_*^2}}. \tag{63}$$

Applying the same procedure to circuit II, we can derive the current flowing on the DL region of the disk and the current density flowing from the BH magnetosphere into the disk as follows,

$$I_{DL} = -I_0\frac{(1+q)\,D\left(a_*,\xi,n\right)}{2}, \quad \xi_S < \xi < \infty \tag{64}$$

$$j_{DL}\left(a_*,\xi,n\right) = \frac{1}{2\pi\left(\varpi\rho/\sqrt{\Delta}\right)_{\theta=\pi/2}}\frac{dI_{DL}}{dr}$$

$$= -\frac{j_0(1+q)C(a_*,\xi)}{2}\frac{\partial D(a_*,\xi,n)}{\partial\xi}, \qquad \xi_S < \xi < \infty, \tag{65}$$

where $C\left(a_*,\xi\right)$ is expressed by equation (63) and $D\left(a_*,\xi\right)$ is expressed by

$$D\left(a_*,\xi,n\right) = \frac{\xi^{-n+2}\chi_{ms}^2\sqrt{1+\xi^{-2}\chi_{ms}^{-4}a_*^2+2\xi^{-3}\chi_{ms}^{-6}a_*^2}}{\xi_S^{n+1}\left(\xi^{3/2}\chi_{ms}^3+a_*\right)\sqrt{1+\xi^{-2}\chi_{ms}^{-4}a_*^2-2\xi^{-1}\chi_{ms}^{-2}}} \tag{66}$$

By using equations (58) and (59) we have the current densities j_{BZ}^H and j_{MC}^H varying with the angle θ on the horizon for $a_* = 0.9$ and the different values of n, and those for $n = 5.5$ and the different values of a_* as shown in Figures 7 and 8, respectively.

By using equations (62) and (65) we have the current densities j_{MC}^D and j_{DL}^D varying with the radial parameter ξ on the disk for $a_* = 0.9$ and the different values of n, and those for $n = 5.5$ and the different values of a_* as shown in Figures 9 and 10, respectively.

From Figures 7—10, we find that the distribution features of the above current densities depend on both the power-law index n and the BH spin a_*. The current densities j_{BZ}^H and j_{DL} are always positive, while j_{MC}^H and j_{MC}^D might change their signs in some cases, which correspond to the corotation magnetic surface (CRMS) as defined in W03b.

6 DISCUSSION

In this paper we discuss and compare the three powers of extracting energy magnetically by considering the restriction of the screw instability to the configuration of the magnetic field in BH magnetosphere. These powers are derived by using circuit I for the BZ and MC processes and circuit II for the DL process. It turns out that the DL power is generally less than the BZ and MC powers, and it is comparable with the two powers for the BH spin a_* approaching unity. By using the conservation laws of energy and angular momentum we discuss the radiation flux from a quasi-steady thin disk around a Kerr BH. Although the situation at the boundary between the MC and DL regions is not very transparent, we obtain the solution for the radiation by resolving the equation from the conservation laws. By using circuits I and II we discuss the distribution of the currents and the current densities on the BH horizon and on the disk for the three energy mechanisms.

Recently Li (2000b) presented a model for extracting clean energy from a Kerr BH surrounded by a dense plasma torus, and argued that this model may be relevant to GRBs, provided that the magnetic field is strong enough. Li argued that the baryonic contamination from the plasma in the torus is greatly suppressed by the magnetic confinement. In fact, the problem of the baryonic contamination can be also suppressed by the closed field lines in the MC region in our model, provided that the magnetic field is strong enough. Inspecting equations (12) and (19), and Figure 4, we find that the maximum of the BZ power could be in the following range,

$$0.05P_0 < P_{BZ}^{\max} < 0.1P_0, \tag{67}$$

which implies that $P_{BZ}^{\max}$ could attains $\sim 10^{51} erg \cdot s^{-1}$ for $M = 7M_\odot$ and $B_4 = 10^{11}$. Although $P_{BZ}^{\max}$ seems much more than needed for powering the beamed GRBs, we can adjust the value of P_{BZ} to fit the beamed GRBs by taking the adequate values of M and B_4 in our model. In addition, the DL process discussed in this paper could be another energy mechanism for extracting clean energy to GRBs. We shall apply this model to GRBs and present a detailed calculation in the future work.

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References

Bateman G. *MHD Instabilities*, 1978, (Cambridge: The MIT Press)
Blandford R. D., 1976, MNRAS, 176, 465
Blandford R. D., & Znajek R. L. 1977, MNRAS, 179, 433
Blandford R. D. 1999, in Sellwood J. A., Goodman J., eds, ASP Conf. Ser. Vol. 160, Astrophysical Discs: An EC Summer School, Astron. Soc. Pac., San Francisco, p.265
Gruzinov A. 1999, astro-ph/9908101
Kadomtsev B. B. 1966, Rev. Plasma Phys., 2, 153
Lee H. K., Wijers, R. A. M. J., & Brown, G. E., 2000, Phys. Rep., 325 83
Li L. -X. 2000a, ApJ, 531, L111
Li L. -X. 2000b, ApJ, 544, L375
Li L. -X. 2002a, ApJ, 567, 463(L02)
Li L. -X. 2002b, A&A, 392, 469
MacDonald D., & Thorne K. S. 1982, MNRAS, 198, 345 (MT82)
Moderski R., Sikora M., Lasota J.P., 1997, in Ostrowski M., Sikora M., Madejski G., Belgelman M., eds, *Relativistic Jets in AGNs.* Uniw. Jagiellonski, Krakow, p.110

Novikov I. D., & Thorne, K. S., 1973, in *Black Holes*,
ed. Dewitt C, (Gordon and Breach, New York) p.345

Page D. N., Thorne K. S., 1974, ApJ, 191, 499

Rees M. J., 1984, ARA & A, 22, 471

Thorne K. S., Price R. H., & Macdonald D. A. 1986, *Black Holes: The Membrane Paradigm*,
Yale Univ. Press, New Haven

Tomimatse A., Matsuoka T., & Takahashi M. 2001, Phys. Rev. D64, 123003

Wang D.-X., Lei W.-H, & Xiao K., 2002a, ApJ, 580, 358

Wang D.-X., Xiao K., & Lei W.-H. 2002b, MNRAS, 335, 655 (W02)

Wang D.-X., Lei W.-H., & Ma R.-Y. 2003a, MNRAS, 342, 851 (W03a)

Wang D.-X., Ma R.-Y., Lei W.-H., & Yao G.-Z., 2003b, ApJ, 595, 109 (W03b)

Wang D.-X., Ma R.-Y., Lei W.-H., & Yao G.-Z., 2004, ApJ 601,1031(W04)

Wilms J. et al. 2001, MNRAS, 328, L27